The result of case II may be stated in the

THEOREM: *Four concyclic points* A_1, B_1, C_1, D_1 *on an equilateral hyperbola form four triangles* $B_1C_1D_1$, $C_1D_1A_1$, $D_1A_1B_1$, $A_1B_1C_1$, *whose orthocenters* A_2, B_2, C_2, D_2 *are also concyclic, and in central symmetry with* $A_1B_1C_1D_1$, *with respect to the center of the hyperbola. The orthocenters of* $B_2C_2D_2$, $C_2D_2A_2$, $D_2A_2B_2$, $A_2B_2C_2$ *coincide in the same order with* A_1, B_1, C_1, D_1.

The fact that A_2, B_2, C_2, D_2 are concyclic is independent of the fact that the points $A_1B_1C_1D_1$ are on an equilateral hyperbola. Necessary and sufficient condition is that A_1, B_1, C_1, D_1 are concyclic. It is always possible to pass an equilateral hyperbola through the points of a proper quadrangle. In fact, the pencil of conics through such a quadrangle intersects the line at infinity in an involution. Any finite point joined to this involution determines an involutory pencil, which contains at least one rectangular pair whose directions are parallel to the asymptotes of the corresponding hyperbola of the pencil. This hyperbola is therefore equilateral.

The foregoing theorem concerning a concyclic quadrangle is not new. It was first stated without proof by Steiner in a foot-note.[1] A purely geometrical proof was first given by Heinen.[2] Later on an analytic proof was published by Greiner.[3]

The question whether A_{n+1}, B_{n+1}, C_{n+1}, D_{n+1} may coincide in the same order, say, with $B_1D_1A_1C_1$ must be answered in the negative, if only real solutions are considered. According to (5) and the three similar equations such a condition would give for α_1, β_1, γ_1, δ_1 fractional values of $i\pi$, and, therefore no proper quadrangle on the hyperbola. Coincidences of less than four points may occur, but as they seem not of sufficient geometric interest, they will not be considered in this note.

REMARKS ON A PREVIOUS ARTICLE.

By NATHAN ALTSHILLER, University of Oklahoma.

In connection with my article "On the I-centers of a Triangle"[1] Prof. J. W. Clawson kindly calls my attention to the fact that the propositions (11) and (13) are known. They were proved in 1906 by the well-known Belgian mathematician, Prof. J. Neuberg, of the University of Liege.[5] Prof. Neuberg states in his article that these propositions were published before without proof.[5]

I arrived at these results in the early summer of 1917 while giving a course in "College Geometry" at the University of Oklahoma, Summer Session. The library facilities at my command were inadequate for a satisfactory biblio-

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[1] *Annales de Mathématiques Pures Appliquées*, Vol. 19, p. 43 (1828).

[2] *Journal für die reine und angewandte Mathematik*, Vol. 3, p. 291 (theorem 9) (1828).

[3] *Archiv der Mathematik und Physik*, Vol. 60, p. 184 (1877).

[4] This MONTHLY, June, 1918, pp. 241–246.

[5] *Mathesis*, 1906, pp. 14–17.

[6] These theorems were also discovered independently by J. V. Morley and published as problems in Volume 24 of this MONTHLY, pages 124 and 430.

graphical investigation. It would seem however that these properties are not well known, since just recently the theorem (11) has been proposed for proof in as serious a journal as the *Nouvelles Annales de Mathématiques*.[1]

It is noteworthy that the sides of the rectangle (11) are proved by Prof. Neuberg to be parallel to the bisectors of the angles formed by the diagonals of the quadrilateral; V. Thébault states that these sides are equally inclined on the sides of the given quadrilateral, while in my paper they are shown to be parallel to the lines joining the mid-points of the arcs subtended by the opposite sides of the given quadrilateral on its circumcircle (compare sections 10, 11). Prof. Clawson adds that they are parallel to the bisectors of the angles formed by any pair of opposite sides of the given quadrilateral.

Thus comes to light an interesting and rather involved property of the inscriptible quadrilateral, worthy perhaps of a direct proof.

BOOK NOTICES.

Edited by W. H. Bussey, University of Minnesota.

A circular advertising the new book on *Unified Mathematics* by Karpinski, Benedict and Calhoun says that "particular attention is paid to problems dealing with projectiles, and the 'mil,' the artillery unit of angular measurement, is carefully explained." But the reader will look in vain for the word "mil" in the index of the book; and he will look in vain in the paragraphs on angles and angular measurement where the degree and radian are defined. However if he is bound to know what a "mil" is and searches further he will be rewarded when he finds Ex. 11 on page 114 which reads: "In the artillery service angles are measured in 'mils'; a 'mil' is defined as 1/6,400 of a complete revolution. Compute the value in radians of one mil." Of course all this is in no sense a real criticism of the book, which was not written primarily for men of the S. A. T. C. The book contains 522 pages. It is supposed to be a course in elementary mathematics adapted to the needs of the freshmen students in the ordinary college or technical school course. According to the preface the material includes the work commonly covered in the past in separate courses in college algebra, trigonometry and analytical geometry. But there is no chapter on Permutations, Combinations and the more simple elements of Probability, and there is no mention of these topics in the index. As one might expect from the fact that Prof. Karpinski is known to be interested in the history of mathematics, the book abounds in historical notes and references. The book is published by D. C. Heath and Co.

Unified mathematics seems to be making its way. In addition to the book just mentioned, several other books on correlated mathematics have recently been published or are about to be published.

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[1] V. Thébault, *N. A. M.*, August, 1917, p. 319.